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Serially Concatenated Multilevel Coding

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Abstract

Multilevel coding is an important approach for constructing powerful coded modulation schemes with high bandwidth efficiency. We consider multilevel codes as inner component codes for a serial concatenation with an outer convolutional code in this work. The performance of such a coding scheme depends among others on the modulation alphabet and the mapping of bits onto the symbols, as well as on decoding strategies and the length of each transmitted frame. The influence of these parameters is considered in this work whereby we consider transmission over both the AWGN as well as the Rayleigh fading channel.

1. INTRODUCTION

The serial concatenation of short codes separated by an interleaver is an efficient method to construct long and powerful codes. The achievable coding gain can be quite large whereby the decoding effort can be kept relatively small when the component codes are decoded separately. The result of this decoding method can be improved by carrying out the whole process iteratively [1].

For communicating at a high spectral efficiency we use a multilevel code (MLC) [2]. The MLC will be embedded as inner code in a serial concatenation with an outer convolutional code (CC) and a random interleaver in between. For conventional serial concatenated CCs (*SCCC*), the serial concatenation of short CCs is used to achieve large coding gains using iterative decoding methods [1]. A similar behavior can be expected for the serially concatenated coded modulation scheme. We focus on binary CCs as component codes of the MLC as well as for the outer code. In the following we refer to this construction as a serially concatenated MLC or *SCMLC* for short.

We investigate some aspects in the design of MLCs and their impact on the performance of the SCMLC. Beside the choice of the level codes and the modulation alphabet, the way the used alphabet is partitioned into subsets according to the binary labeling of its symbols is important [3]. We will investigate this influence using two different partitioning strategies.

Since the decoding process is carried out iteratively including an iterative MLC decoder it is possible to apply various decoding strategies. Special strategies are presented here to show the influence on the decoding result.

The blocksize, i. e. the number of information bits that generate one frame of code symbols, is also an important parameter in the SCMLC scheme. This is because the length of the interleaver between the inner and the outer code is directly related to this parameter. The influence of varying blocksizes on the code performance will also be a topic of our considerations.

In Section 2, there will be a brief introduction into multilevel coding and the partitioning of modulation alphabets. The complete SCMLC coding scheme is described in Section 3, where also a short description of the decoding process is given. In Section 4, simulation results will be presented and interpreted with respect to the chosen partitioning strategies, decoding strategies and interleaver lengths. This work ends with a summary of the insights gained about SCMLCs in the conclusions in Section 5.

2. MULTILEVEL CODING

Multilevel coding is an important approach for constructing codes in the Euclidean space and was first introduced in [2]. This coding method is a very flexible construction to adapt forward error correction schemes to higher order modulation alphabets by usage of parallel component codes, the so called *level codes*. The basic idea is that m level codes choose symbols out of an m-ary alphabet $\mathcal{X}$ whereby each level determines only one of the m address bits of a symbol. Thus the labeling of a partitioned symbol constellation is protected by m parallel level codes.

2.1. Multilevel Encoder

In the multilevel encoder we use binary CCs to protect the address string of each modulation symbol. The encoder is depicted in Figure 1. The sequence of infor-

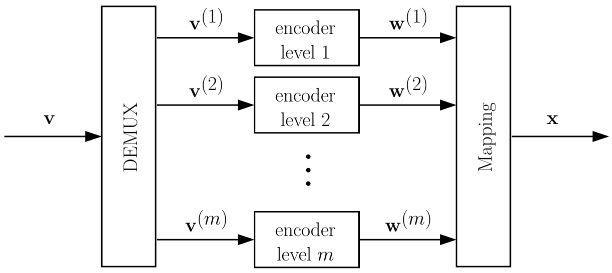

Figure 1: MLC encoding scheme

mation bits $\mathbf{v}$ is demultiplexed to the m level encoders thus resulting in the sequences $\mathbf{v}^{(j)}$, $j = 1, \dots, m$. These sequences are binary vectors $(v_1^{(j)} v_2^{(j)} \dots v_{l_j}^{(j)})$ of lengths l_j depending on the rates $r^{(j)}$ of the level codes. These rates and the lengths l_j are chosen such that all sequences of codebits $\mathbf{w}^{(j)} = (w_1^{(j)} w_2^{(j)} \dots w_l^{(j)})$ have the same length l. The level encoder is depicted in Figure 2 in detail. The input sequence $\mathbf{v}^{(j)}$ is encoded and the possibility of puncturing and interleaving ($P^{(j)}$ and $\pi^{(j)}$) is offered to generate the sequence $\mathbf{w}^{(j)}$. The rate of the punctured level code will be denoted $r_p^{(j)}$.

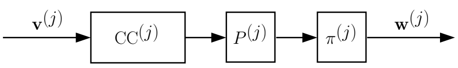

Figure 2: Detailed encoder for level j

The mapper which follows the level encoders takes one bit from each sequence $\mathbf{w}^{(j)}$ to select an element of the alphabet $\mathcal{X}$ whereby the codebits of level one are the most significant bits and those of level m are the least significant bits. At time instant i we therefore have the label string $(w_i^{(1)} \cdots w_i^{(m)})$ selecting the ith output symbol $x_i \in \mathcal{X}$. Thus, the output sequence $\mathbf{x} = (x_1 x_2 \dots x_l)$ with a *frame length* of l symbols is generated which will then be sent over the channel.

For the design of MLCs there exist many approaches [3]. One approach is to optimize the asymptotic behavior of the code. For the AWGN channel this results in a preferably high minimum squared Euclidean distance of the MLC. Together with a maximization of the total code rate, one has to apply the *balanced distance rule* where for each level the product of free distance of the code and the minimum squared Euclidean distance of all subsets in the partitioning level should be equal.

For the Rayleigh fading channel, the asymptotic code performance is determined by its minimum Hamming distance which therefore has to be maximized. The *Hamming distance rule* hence chooses level codes with equal minimum Hamming distances under the constraint of a high overall code rate.

Another approach in designing MLCs is the *error probability rule*. The level codes are chosen in a way, that all of them are contributing identically to the total bit error rate of the MLC. This method is not restricted to high E_b/N_0 and can furthermore be applied to both, the AWGN and the Rayleigh fading channel.

2.2. Partitioning Strategies

The choice of the modulation alphabet and the mapping of the address bits onto the elements of this alphabet has main influences on the distance properties of multilevel codes. The mapping can be seen as a *partitioning* of the modulation alphabet.

The alphabet $\mathcal{X}$ with $|\mathcal{X}| = 2^m$ elements $\in \mathbb{C}$ is partitioned in the first level into two disjoint subsets according to whether the most significant bit of the address vector of the symbols is 0 or 1. These subsets can be further partitioned according to the other $m-1$ bits of the address vector until all elements of $\mathcal{X}$ are partitioned into $|\mathcal{X}|$ subsets each with only one symbol. For each partitioning level j, we calculate the minimum squared Euclidean intra-subset distance δ_j^2. With $\delta_0^2 = \delta^2$, the minimum squared Euclidean distance of $\mathcal{X}$, and $\delta_m^2 = \infty$ we have

$$\delta_0^2 \le \delta_1^2 \le \dots < \delta_m^2. \tag{1}$$

The partitioning strategies which are in the focus of our investigations are the *Ungerböck* partitioning [4] and the *Gray* labeling. The terms labeling and partitioning strategy are used equivalently here.

The Ungerböck partitioning maximizes the minimum squared Euclidean distance δ_j^2 in each level. The increase of δ_j^2 should be as large as possible from level to level. This can be achieved by partitioning adjacent symbols into two different subsets. Thus, we have $\delta_0^2 < \delta_1^2 < \dots < \delta_m^2$.

The second partitioning strategy which is considered in this work is the Gray labeling. Hereby, the label strings are mapped to the symbols such that the labelings of two nearest neighbors differ only in one digit. With this strategy no increase of the level distances δ_j^2 is achieved, we rather have $\delta_j^2 = \delta^2$ for $0 \leq j < m$.

As an example, in Figure 3 the Ungerböck and Gray partitioning strategies for a 16QAM signal constellation are shown.

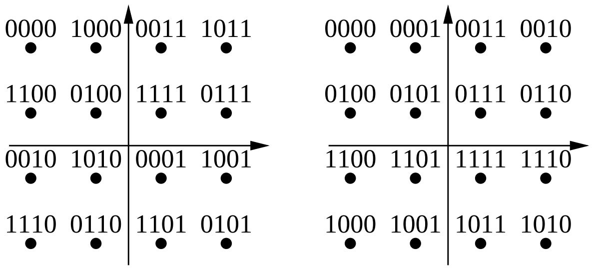

Figure 3: Ungerböck (left) and Gray (right) partitioned 16QAM constellation

2.3. Multilevel Decoder

For decoding MLCs, a method was already presented in [2], i. e. the *multistage decoder* (MSD). In this decoder each level code is decoded separately. The calculated log-likelihood ratios or L-values on the codebits of level j, denoted $\tilde{\mathbf{w}}^{(j)} = \left(\tilde{w}_1^{(j)} \dots \tilde{w}_l^{(j)}\right)$, are passed to all subsequent levels as a-priori information. The advantage of this decoding technique is, that the decoding complexity is kept small since the level codes are decoded separately. An overall decoding of the complete MLC would be very extensive and the reduction of complexity for MSD is quite large. However, the drawback of this decoding technique is that no a-priori information is available for the first level. With several repetitions of the MSD decoding step we can overcome this drawback and improve the performance of the MLC. The resulting decoder is a so called *iterative* MSD or iMSD [5], [6], [7]. The main idea is that the L-values which are calculated for the codebits of each level during the first iteration are used as a-priori information in the following decoding step. Thus, after the first iteration, for all levels there is a-priori information from all other levels available. For a particular level j and iteration number ν these a-priori values are $\tilde{\mathbf{w}}_\nu^{(k<j)}$ from levels $k < j$ of the current iteration and $\tilde{\mathbf{w}}_{\nu-1}^{(k>j)}$ from levels $k > j$ of the previous iteration $\nu - 1$. This improves the performance of the code significantly. In Figure 4 the iMSD decoding scheme can be seen. Compared with a maximum-likelihood decoder of the overall MLC, the complexity of the iMSD method is by far smaller as already mentioned. Additionally in [7] it was shown, that the performance of the iMSD is quite close to the maximum-likelihood decoding result.

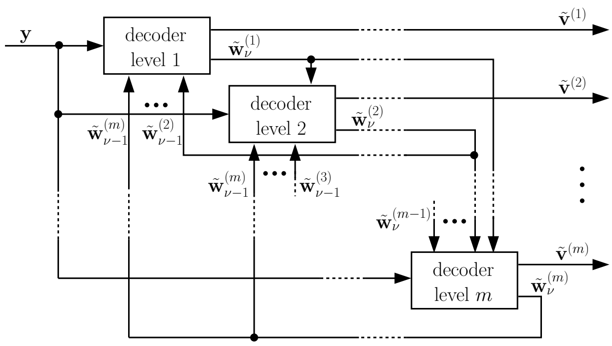

Figure 4: iMSD scheme

3. SCMLC ENCODING AND DECODING SCHEME

The encoder of the SCMLC scheme is shown in Figure 5. Assume that the information bits are the elements of the sequence $\mathbf{u} = (u_1 u_2 \dots u_k)$ of *block length* k with $u_i \in GF(2)$. This is the input of the outer convolutional encoder. The sequence of encoded bits of length n at the encoder's output is denoted $\mathbf{v} = (v_1 v_2 \dots v_n)$. Hence, the rate of the outer code is $r = k/n$ and if puncturing is applied, then the rate is denoted r_p. After interleaving these codebits using a random interleaver we get the sequence $\mathbf{v}'$ which is the sequence of information bits for the inner MLC with m levels. The inner encoder generates the sequence $\mathbf{x} = (x_1 x_2 \dots x_l)$ with a frame length of l symbols. Hereby the elements x_i are chosen from the alphabet $\mathcal{X}$ with $|\mathcal{X}| = 2^m$. Therefore, the bandwidth efficiency η of the SCMLC can be calculated as

$$\eta = k/l \quad \text{[bit/s/Hz]}, \tag{2}$$

when ideal Nyquist pulse shaping is assumed. This means that we transmit η information bits per modulation symbol.

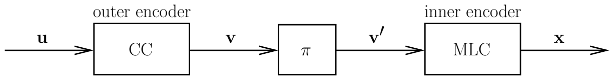

Figure 5: SCMLC encoding scheme

The sequence $\mathbf{x}$ is transmitted over the channel. For the AWGN channel the noise vector $\mathbf{n} = (n_1 n_2 \dots n_l)$, i. e., a sequence of Gaussian distributed complex-valued random variables, is added component wise to the sequence $\mathbf{x}$. Thus, the elements of the channel output $\mathbf{y} = (y_1 y_2 \ldots y_l)$ can be calculated as $y_i = x_i + n_i$, $y_i \in \mathbb{C}$. For the Rayleigh fading channel with a sequence of Rayleigh distributed complex-valued fading coefficients $\mathbf{a} = (a_1 a_2 \ldots a_l)$ we have the channel output $y_i = a_i x_i + n_i$.

The SCMLC decoder can be seen in Figure 6. It is very similar to the decoding scheme of an SCCC [1]. First, we have an iMSD decoder for decoding the MLC. The L-values on the inner information bits which were calculated there, i. e. $\tilde{\mathbf{v}}^{(1)} \ldots \tilde{\mathbf{v}}^{(m)}$, are serialized. Except for the first iteration, the MLC decoder can make use of a-priori information $\mathbf{L}_a$ stemming from the decoder of the outer CC. To avoid feeding back the information from the outer decoder to itself we subtract $\mathbf{L}_a$ from the decoding result of the inner decoder which results in the log-likelihood ratios $\mathbf{L}_e$. These L-values are used as soft input for the outer decoder after deinterleaving. The output of the outer decoder are the L-values on the codebits of the outer CC, denoted $\mathbf{L}_c$. The extrinsic information on these bits is obtained by subtracting the input of the outer decoder from the output $\mathbf{L}_c$. These L-values are interleaved and used as a-priori information $\mathbf{L}_a$ about the inner information bits in a new decoding step. In the outer decoder as well as in the level decoders of the inner decoder the BCJR algorithm performs symbol by symbol a posteriori probability (s/s-APP) decoding.

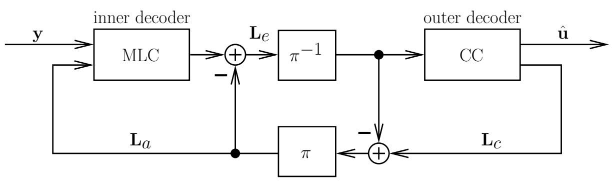

Figure 6: SCMLC decoder scheme

The whole decoding process can be carried out iteratively. We investigate different decoding strategies (DSs) since it is possible to vary the number of iMSD iterations which are carried out per overall iteration. For example, if we apply 5 overall iterations and three iMSD iterations in each of them, we denote this decoding strategy DS_5(3-3-3-3-3) or $\mathrm{DS}_5(3^5)$ for short. If we carry out three iMSD iterations in the first overall and then only one (MSD) in the four remaining overall iterations, this will be denoted DS_5(3-1-1-1-1) or $\mathrm{DS}_5(3\text{-}1^4)$.

4. SIMULATION RESULTS

The impact of the chosen partitioning strategy, the decoding strategy and the length of the interleaver on the performance of SCMLCs is demonstrated by simulation results in this section. The simulations are carried out using an outer mother code of rate $r = 1/2$ with generator matrix $\mathbf{G}(D) = \big(1 + D^2, 1 + D + D^2\big)$ [8]. As inner codes we use MLCs with $m = 4$ levels together with a 16QAM alphabet whereby the same mother code as for the outer code is used for the level codes. The puncturing in each level is chosen such that the resulting MLC has rate 3/4. The outer code is punctured to rate $r_p = 2/3$ thus achieving a spectral efficiency of $\eta = 2.0$ bit/symbol for the SCMLCs. We communicate over the AWGN channel and the Rayleigh fading channel and transmit frames with $l = 504$ symbols corresponding to a block length of $k = 1002$ information bits. We achieve the above mentioned efficiency only approximately due to termination bits for all component encoders.

First, we investigate the influence of the partitioning strategy for a given modulation alphabet on the performance of the SCMLC. We consider the 16QAM alphabet with Ungerböck (U) and Gray (G) partitioning. For the AWGN channel we have the codes SCMLC1 (SCMLC2) for $16\mathrm{QAM}_U$ ($16\mathrm{QAM}_G$) and for the Rayleigh fading channel we have analogously SCMLC3 and SCMLC4. The parameters of these codes can be seen in Table 1 and Table 2. The MLCs are designed according to the error probability rule, i. e., all levels have nearly the same contribution to the overall error rate. The decoding was done by applying 10 overall iterations, each with just one inner multistage decoding, i. e., $\mathrm{DS}_{10}(1^{10})$. The performance of these codes is shown in Figure 7 in terms of the BER. For both channels it can be observed that the codes with Gray labeling (SCMLC2 and SCMLC4) perform much better than those with Ungerböck labeling. Furthermore, SCMLCs may tend to an error floor [6]. The beginning of this floor can be seen for code SCMLC4 already at a BER of 10^{-5}. For SCMLCs with Ungerböck labeling there may be also an error floor. Due to a larger minimum squared Euclidean distance in this case, the error floor may presumably appear only at a much lower BER.

Now we investigate the influence of the chosen decoding strategy on the performance of the SCMLCs. We consider Ungerböck and Gray partitioning for the 16QAM constellation and transmission over the AWGN channel. In Figure 8 the BER curves for SCMLC1 using Ungerböck labeling and for SCMLC2 using Gray labeling are shown. It can be observed that for SCMLC1 both $\mathrm{DS}_{10}(1^{10})$ and $\mathrm{DS}_{10}(1^9\text{-}4)$ have the lowest BER, followed by $\mathrm{DS}_{10}(4\text{-}1^9)$ and the worst performance is achieved by $\mathrm{DS}_{10}(4^{10})$. Thus, for the Ungerböck partitioning it is beneficial for the performance of the SCMLC to apply only one iteration of the inner decoder (MSD). No improvement of the BER can be achieved by applying iMSD in the last overall iteration, i. e., $DS_{10}(1^9\text{-}4)$. However, if in the first overall iteration iMSD is used, the performance degrades. This is because the absolute values of log-likelihood ratios with wrong signs of the inner decoder increase with iMSD iterations. Therefore, the outer decoder can only hardly correct these errors resulting in a higher BER. If additionally in any other overall iteration iMSD is used, this effect is intensified and the performance is significantly decreased. For SCMLC2 we do not have such a large difference of the BERs applying different decoding strategies. A reason for this behavior may be that each bit of a constellation symbol experiences nearly the same equivalent channel [3]. Therefore, iterative decoding between the levels does not help much. The main contribution to the performance of the SCMLC is due to the iterations between the inner and the outer code.

In the following, we analyze the effect of varying the block length and therefore the length of the interleaver between the inner and the outer code. We focus on the Gray labeled 16QAM constellation for AWGN and for Rayleigh fading channel, i. e., SCMLC2 and SCMLC4, and apply decoding strategy $DS_{10}(1^{10})$. The block length is increased by a factor of 10 and of 100 compared to the original block length of $k = 1002$ information bits. The BERs of these codes are shown in Figure 9. For both channels, the performance improves significantly for increasing the block length since incorrect decisions of the inner decoder are spread and thus the probability of large error bursts for the outer code is decreased.

Please note that the minimum E_b/N_0 which is required for an error free transmission over the AWGN channel at $\eta = 2.0$ bit/symbol with 16QAM is ∼2.1 dB. Using the largest block length for SCMLC2, the gap to this limit is about 1.2 dB at a BER of 10^{-5}.

5. CONCLUSIONS

In this work, serially concatenated multilevel codes (SCMLCs) were presented. We investigated the influence of design parameters for this coding scheme. We saw that for the AWGN and the Rayleigh fading channel the codes using a Gray partitioned modulation alphabet perform much better than those using Ungerböck partitioning. Furthermore, the applied decoding strategy is important. For Ungerböck partitioning, the strategy which applies only MSD and the one which applies iMSD in the last overall iteration perform best. The iMSD decoding in other iterations leads to increased error rates. For the Gray partitioned alphabet, there are no big differences between different strategies. Additionally, we varied the interleaver length. With increasing length, the SCMLC performance was significantly improved.

Acknowledgments

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SCMLC1		SCMLC2	
outer rate $r_p = 2/3$		outer rate $r_p = 2/3$	
$k = 1002$ bits $l = 504$ symbols		$k = 1002$ bits $l = 504$ symbols	
level j	$r_p^{(j)}$	level j	$r_p^{(j)}$
1	1/2	1	6/8
2	6/8	2	5/8
3	7/8	3	7/8
4	7/8	4	6/8
$16QAM_U$		$16QAM_G$	

Table 1: SCMLC parameters for AWGN

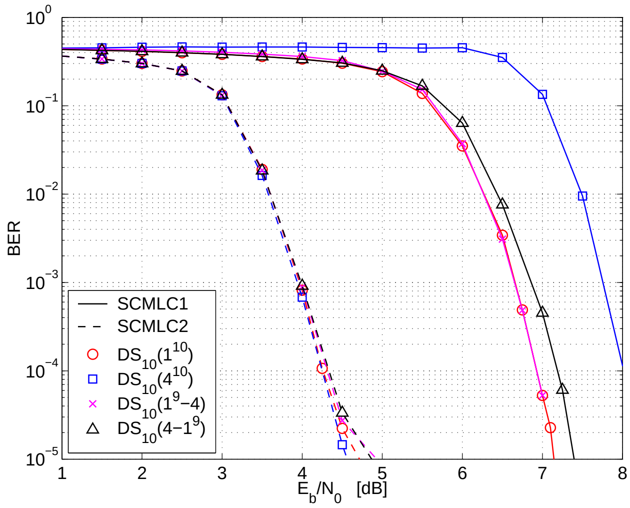

Figure 8: SCMLC1 ($16QAM_U$) and SCMLC2 ($16QAM_G$) decoding strategies, AWGN channel

SCMLC3		SCMLC4	
outer rate $r_p = 2/3$		outer rate $r_p = 2/3$	
$k = 1002$ bits $l = 504$ symbols		$k = 1002$ bits $l = 504$ symbols	
level j	$r_p^{(j)}$	level j	$r_p^{(j)}$
1	1/2	1	5/8
2	6/8	2	6/8
3	7/8	3	6/8
4	7/8	4	7/8
$16QAM_U$		$16QAM_G$	

Table 2: SCMLC parameters for Rayleigh fading

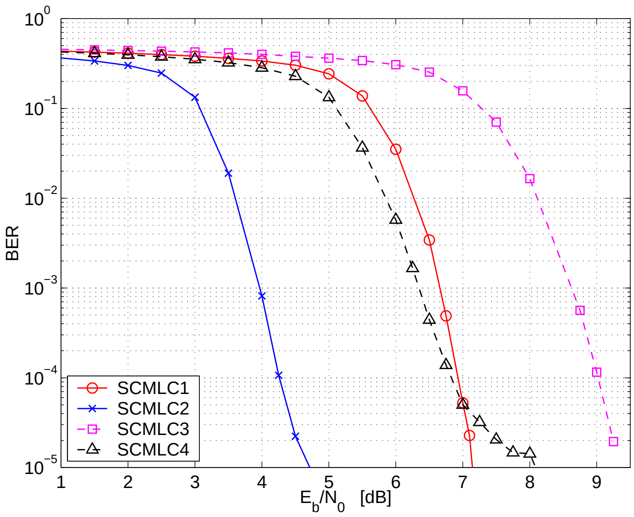

Figure 7: 16QAM partitioning strategies

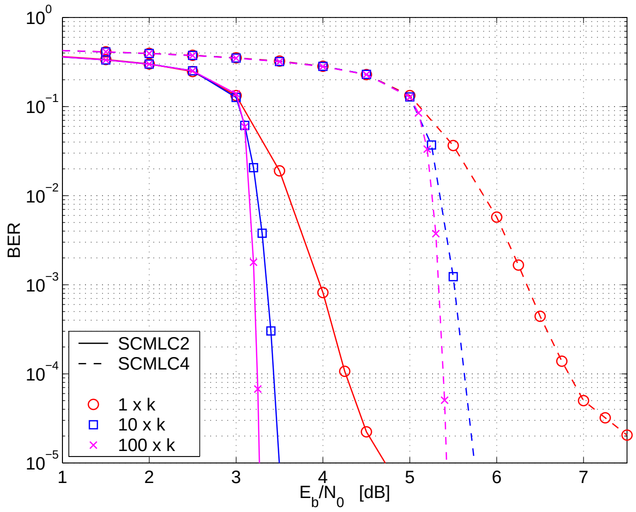

Figure 9: SCMLC2 and SCMLC4 interleaver sizes