

PAPR Reduction Methods for Noncoherent OFDM-MFSK

Matthias Wetz, Werner G. Teich, Jürgen Lindner
Department of Information Technology, University of Ulm
Albert-Einstein-Allee 43, D-89081 Ulm, Germany
Email: {matthias.wetz, werner.teich, juergen.lindner}@uni-ulm.de

Abstract—MFSK (M-ary frequency shift keying) can be combined with OFDM (orthogonal frequency division multiplexing) by grouping the subcarriers into groups of M and applying MFSK modulation to each of these groups. The detection of such an OFDM-MFSK modulation scheme can be noncoherent and it is very robust against Doppler shift and Doppler spread. The noncoherent detection of OFDM-MFSK allows an arbitrary phase choice for all subcarriers in the transmitter. This degree of freedom can be exploited to reduce the PAPR (peak-to-average power ratio), which is a well known problem for multicarrier transmission schemes. In this paper we compare several PAPR reduction schemes and apply them to OFDM-MFSK. A new PAPR reduction algorithm is introduced, which adjusts the subcarrier phases of an OFDM symbol to obtain a low PAPR. The effects on the spectrum of the OFDM-MFSK signal in the presence of a nonlinear transmitter are considered and the improvements achievable by the various PAPR reduction schemes are shown. Finally the complexity of the presented schemes is compared.

I. INTRODUCTION

It is a well known problem, that multicarrier transmission schemes like OFDM (orthogonal frequency division multiplexing) suffer from a large peak-to-average power ratio (PAPR). The reason for this is, that for certain phase selections of the subcarriers, the superposition of the orthogonal subcarriers leads to very large peaks in the amplitude.

To ensure linear amplification of a signal with a large PAPR, the amplifier has to be operated with a large input back off (IBO), which means that the mean power has to be chosen sufficiently low, leading to a very low efficiency of the amplifier. If the IBO is chosen too small, the signal will be distorted. This leads to an increase in the bit error rate and out of band radiation (OBR). Because the OBR is strictly limited by regulations, this is the more severe effect. Therefore it is necessary to reduce the PAPR by appropriate methods. In this paper we consider PAPR reduction methods for a new combination of OFDM and MFSK (M-ary frequency shift keying), which we call OFDM-MFSK. In OFDM-MFSK the OFDM subcarriers are grouped into groups of M . Like in conventional MFSK modulation, on one subcarrier of each group energy is transmitted whereas to all other subcarriers of this group no energy is allocated. The noncoherent detector decides for the subcarrier where most energy is received, so that the phase of the transmit symbol is arbitrary. The receiver of this scheme does not need any channel state infor-

mation. Furthermore OFDM-MFSK is very robust against fast fading channels. Because the phases of the occupied subcarriers do not affect the transmission, they can be chosen appropriately to reduce the PAPR.

There are many known PAPR reduction techniques for QAM and MPSK modulated multicarrier transmission. A good overview is given in [1]. After introducing the OFDM-MFSK system model we compare some of these reduction techniques in Section IV. We also suggest a new algorithm to adjust the subcarrier phases in order to reduce the PAPR. In Section V we consider the influences of nonlinear distortion on the spectrum of OFDM-MFSK signals.

II. SYSTEM DESCRIPTION

We use a conventional OFDM transmission model for our investigations. The complex baseband representation of the transmit signal $s(t)$ is obtained as a sum of orthogonal subcarriers given by

$$s(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x_n \cdot e^{j2\pi n\Delta f t}, \quad (1)$$

where N represents the number of subcarriers and Δf the subcarrier spacing. x_n represents the elements of the OFDM symbol vector $\mathbf{X} = [x_0, \dots, x_{N-1}]^T$. In the case of noncoherent OFDM-MFSK the elements of \mathbf{X} are divided into groups of M and, depending on the transmit data, one element i of each group is chosen so that $|x_i| = 1$ and $x_{n \neq i} = 0$ for the rest of this group. This means that for each OFDM symbol N/M subcarriers are occupied. This scheme is illustrated in Fig. 1 for OFDM-4FSK on which we will also focus in the rest of this paper. Solid arrows indicate occupied subcarriers whereas dashed arrows indicate empty subcarriers. We use $N = 256$ subcarriers and a discrete time domain representation, so that the transmit signal can be obtained by an inverse fast Fourier transform (IFFT).

III. THE PAPR OF MULTICARRIER SIGNALS

The PAPR of an OFDM symbol is defined as the square of the maximum amplitude divided by the mean power. If

$$\|s\|_{\infty} = \max_t |s(t)| \quad (2)$$

is the maximum amplitude and

$$\|s\|_2^2 = \frac{1}{T_s} \int_0^{T_s} |s(t)|^2 dt \quad (3)$$

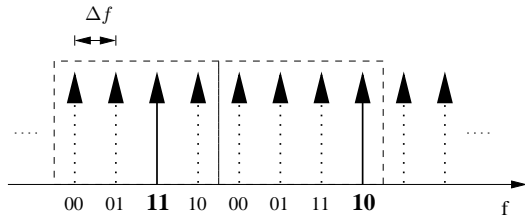


Fig. 1. Principle of OFDM-4FSK modulation. Two data bits can be allocated to each subcarrier. Solid arrows indicate occupied subcarriers. Dashed arrows indicate empty subcarriers.

is the mean power of an OFDM symbol, then the PAPR is defined as [2]

$$\text{PAPR} = \frac{\|s\|_{\infty}^2}{\|s\|_2^2}. \quad (4)$$

We will use a complex baseband notation for our investigation. This is appropriate for high carrier frequencies. The PAPR of the real valued bandpass signal PAPR_r is related to the lowpass PAPR by $\text{PAPR}_r = 2 \cdot \text{PAPR}$ [3].

If all subcarriers are occupied and if we allow all subcarrier phases, the time domain samples of the transmit signal are approximately Gaussian distributed. Without oversampling, the time domain samples are mutually uncorrelated and the probability that the PAPR is below a certain threshold z , i.e. the cumulative distribution function (CDF), can be written as [2]

$$P(\text{PAPR} \leq z) = \text{CDF}(z) = (1 - e^{-z})^N. \quad (5)$$

For large N and if we choose the subcarrier phases randomly, equation (5) is also valid for the case of OFDM-MFSK, where not all subcarriers are occupied. However there is a difference in the maximum PAPR. For the case where all subcarriers are occupied, the maximum amplitude is achieved when all subcarriers add coherently and according to (1) is \sqrt{N} . Due to the normalization in (1), the mean power of such an OFDM symbol is 1. Therefore the maximum PAPR is N . If we now consider OFDM-MFSK, the maximum amplitude is \sqrt{N}/M and the mean power is $1/M$, so that the maximum PAPR becomes N/M .

Accounting to the sampling theorem, bandlimited continuous-time signals can be represented by appropriate discrete-time samples. The continuous-time signal can be obtained by interpolating the discrete-time samples with a lowpass filter. However care has to be taken when calculating the PAPR of the continuous-time signal. Because interpolation can cause higher peaks in the signal amplitude, the PAPR of the continuous-time signal can be significantly larger than the PAPR of the discrete-time signal. This interpolation can be taken into account by using a larger IFFT size and padding the spectrum accordingly with zeros, which results in an oversampled discrete time domain signal. In this paper we use an oversampling factor of eight, which is sufficient to approximate the PAPR of the continuous OFDM signal [1].

IV. PAPR REDUCTION SCHEMES

The largest possible PAPR of N/M for an OFDM-MFSK symbol is obtained when using the same initial phase for all occupied subcarriers. In our case of 256 subcarriers and OFDM-4FSK this results in a PAPR of 64 or 18 dB. A straightforward method to reduce this value is to assign random phases φ_n to all subcarriers. Simulations have shown that allowing only discrete phases $\varphi_n = \{0, \pi\}$ leads to a lower PAPR than allowing continuous phases $\varphi_n \in [0, 2\pi)$. The CDF of the PAPR for these phase selection schemes can be seen in Fig. 2. Most of the OFDM symbols have a PAPR between 6 dB and 10.5 dB. For each possible OFDM symbol there are subcarrier phases which lead to a low PAPR. However it is not an easy task to find the best phase allocation. For OFDM-4FSK with $N = 256$ there are 4^{64} possible OFDM symbols. If we allow only two different phases for each occupied subcarrier, there are still 2^{63} possibilities for each symbol to allocate the subcarrier phases. This means that performing an exhaustive search like in [4] and storing the results in a lookup table is impossible. So we have to find different methods with lower complexity which can reduce the PAPR. For QAM and MPSK modulated OFDM there are many known PAPR reduction schemes. In the following subsections we will apply some of them to OFDM-MFSK and compare their performance and complexity. We will also present a new algorithm which is a trade off between performance and complexity. One advantage of OFDM-MFSK is, that the phases can be chosen arbitrarily so that we do not have to transmit any side information about the selected phases.

A. Selected Mapping

Selected mapping was introduced in [5] for MPSK modulation. It generates several different representations of the same OFDM data symbol by multiplying the OFDM symbol with different phase vectors. The resulting vector with the lowest PAPR is transmitted. To recover the phase information, it is of course necessary to transmit to the receiver as side information which phase vector was used.

We apply this technique to OFDM-MFSK by assigning random phases to the occupied subcarriers. This is done several times and the symbol with the lowest PAPR is transmitted. Simulations have shown, that the best results can be obtained if we use only binary phases, e.g. 0 or π . For noncoherently detected OFDM-MFSK it is not necessary to know the transmit phases, because the phase of the subcarriers has no influence on the detection of the signal. Simulation results for the PAPR using selected mapping are shown in Fig. 2 where the best of two, four, and ten symbols is selected, respectively. Fig. 2 shows that increasing the number of symbols with random phases from which we can choose leads to lower PAPR values. However, the complexity grows as well, because all candidate symbols have to be transformed to the time domain by an IFFT and the PAPR has to be evaluated.

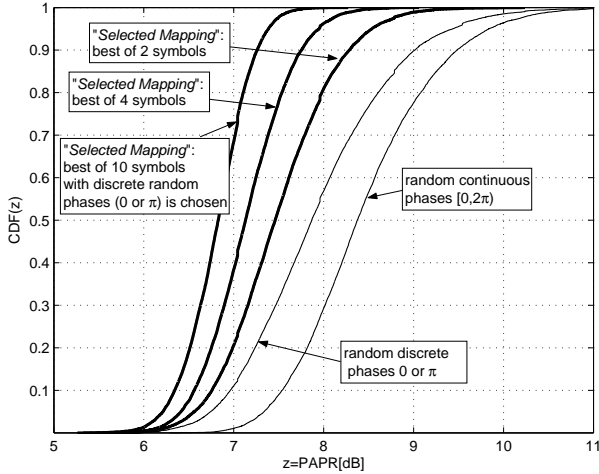


Fig. 2. CDF of the PAPR using OFDM-4FSK modulation for different PAPR reduction schemes with random phase selection. $N = 256$, 8 times oversampling.

It is also possible to limit the PAPR to a certain value for selected mapping by repeating the procedure of assigning random phases to occupied subcarriers until the PAPR is below a certain threshold. However we do not know how often we have to rechoose the phases until the PAPR is below the threshold. Depending on the value of the threshold a very large number of repetitions may be necessary leading to a large complexity and large delay.

B. Time-Frequency Domain Swapping Algorithm

In [3] and [6] an algorithm is described where the PAPR of a multicarrier signal is iteratively decreased by switching between time and frequency domain. This algorithm can also be used to reduce the PAPR of our OFDM-MFSK symbols. In a first step, random phases $\varphi_n \in [0, 2\pi)$ are assigned to all occupied subcarriers. The signal is transformed to the time domain via an IFFT where it is clipped at a certain value related to the extremal value. Typical values are between 75 and 95 percent. An FFT is used to transform the clipped time domain signal back into the frequency domain. As a new signal in the frequency domain, the original signal with the subcarrier phases of the clipped signal is used. This algorithm is repeated until the PAPR is not decreasing any more. While the subcarrier amplitudes stay the same during the algorithm, the PAPR is reduced by adapting the subcarrier phases. The choice of the clipping level (CL) has a great influence on the performance of the algorithm. If the clipping level is chosen close to 100%, the convergence is very slow. On the other hand if it is chosen too low, the algorithm will terminate very soon because the clipping alters the signal too much so that the algorithm may diverge. Fig. 3 shows the results for the PAPR using the time-frequency domain swapping algorithm. The influence of the clipping level is clearly visible. The higher CL the lower the resulting PAPR but because of the slow convergence many iterations

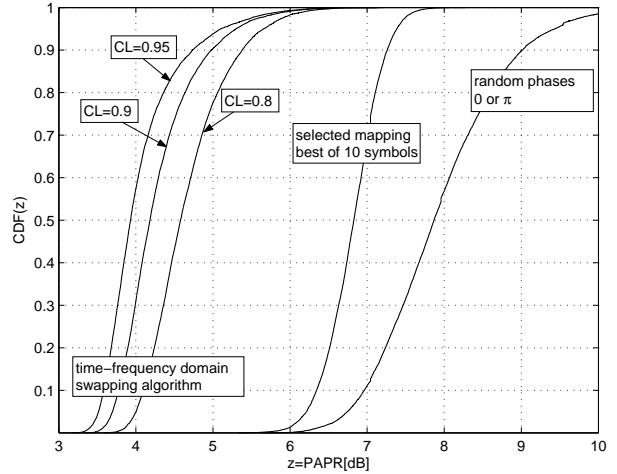


Fig. 3. CDF of the PAPR using OFDM-4FSK modulation for the time-frequency domain swapping algorithm with different clipping levels (CL). $N = 256$, 8 times oversampling.

are needed. Compared to selected mapping, this algorithm yields much lower PAPR values. Most of the values are between 3.5 dB and 6 dB. However, this improvement comes with a significant increase in complexity.

C. Sequential Algorithm

In this subsection we present a new algorithm to reduce the PAPR of OFDM-MFSK symbols. In this algorithm, the subcarrier phases are systematically changed so that the PAPR is reduced. In a first step, the phases of the occupied subcarriers are chosen randomly from the set $\varphi_n = \{0, \pi\}$. After this the symbol is transformed to the time domain and the PAPR is calculated. The algorithm now flips the phases of the subcarriers sequentially, checking if the PAPR of the time domain signal has been reduced after each phase flip. If the phase flip has reduced the PAPR, the flipped phase is used, otherwise the original phase is kept. Simulations have shown that allowing more than two phase steps does not lead to significantly lower PAPR values but the complexity is increased. The flowchart of the sequential algorithm is depicted in Fig.4. As the flowchart shows, there are $N/M + 1$ IFFTs per OFDM symbol necessary for this algorithm, because only occupied subcarriers have to be considered. If we look at the performance of the sequential algorithm, we can see from Fig. 5, that for our parameters, most of the obtained PAPR values are between 5 dB and 7 dB. Compared to selected mapping with ten candidate symbols, the sequential algorithm lowers the PAPR values by about 1 dB. If we use selected mapping with the same complexity as the sequential algorithm, i.e. 65 candidate symbols, the sequential algorithm outperforms selected mapping as can be seen in Fig. 5. If we compare the results to the time-frequency swapping algorithm, we see that it performs significantly better than the sequential algorithm, however the complexity is much larger. Therefore the sequential algorithm can be seen as a trade off between complexity and performance.

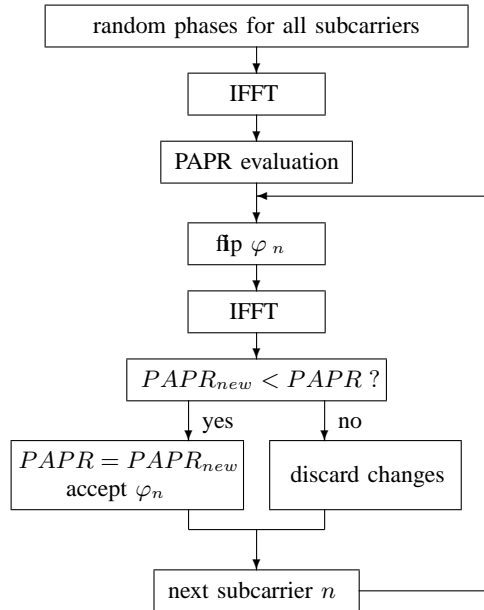


Fig. 4. Flow chart of the sequential algorithm

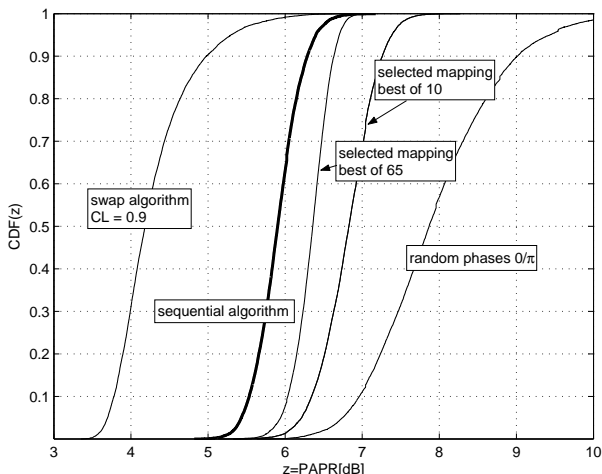


Fig. 5. CDF of the PAPR using OFDM-4FSK modulation for the sequential compared to other algorithms; $N = 256$, 8 times oversampling

D. Complexity Considerations

All of the presented PAPR reduction algorithms have the problem that the phases have to be selected for each OFDM symbol individually. As this has to be done in real time for a real system, the complexity of the algorithms is very important. Table I gives an overview over the complexity of the analysed PAPR reduction techniques. The column of the obtained PAPR values represents the region which contains 99% of the PAPR values. As a measure of complexity the number of necessary Fourier transforms per OFDM symbol is used.

Assigning random phases to occupied subcarriers comes at no extra cost in complexity, however the PAPR values are very large. Using selected mapping with ten candidate symbols increases the complexity to ten IFFTs and an

Reduction algorithm	PAPR values (99%-region)	Complexity
random phases 0 or π	6.0-10.5 dB	1 IFFT
selected mapping (best of 10 symbols)	5.8-7.8 dB	10 IFFTs
selected mapping (best of 65 symbols)	5.7-6.9 dB	65 IFFTs
sequential algorithm	5.1-6.8 dB	65 IFFTs
time-frequency swapping algorithm (CL=0.8)	3.8-6.5 dB	approx. 200 (I)FFTs
time-frequency swapping algorithm (CL=0.9)	3.5-6.2 dB	approx. 800 (I)FFTs

TABLE I

COMPLEXITY AND PERFORMANCE OF THE ANALYSED PAPR REDUCTION TECHNIQUES FOR OFDM-4FSK AND $N = 256$.

evaluation of the PAPR of each symbol but the probability of obtaining large PAPR values is significantly decreased. If we use the sequential algorithm, we can reduce the PAPR values by about one more dB. However we need one IFFT per flipped subcarrier which rises the complexity to 65 IFFTs in our example. Selected mapping with the same complexity leads to larger PAPR values than using the sequential algorithm. The lowest PAPR values are obtained with the time-frequency domain swapping algorithm. For each iteration of the algorithm an IFFT as well as an FFT have to be executed. Depending on the clipping level, the number of iterations is also quite large which leads to a complexity that can hardly be implemented in real systems.

V. INFLUENCE ON THE SPECTRUM

In this Section we investigate the effects of nonlinear distortion on the spectrum of the transmit signal. If the transmit amplifier is operated out of its linear region, i.e. if the IBO is too small, the signal is distorted, which leads to in-band distortion and out of band radiation (OBR). Regulations limit the OBR, so it is necessary to keep it as low as possible. For simulations, the amplifier has been modeled as a soft limiting device. Under certain conditions the behaviour of the transmit amplifier can be modeled in the low pass domain [7].

The spectrum of OFDM with rectangular basic waveforms decays with $\frac{1}{f^2}$. For wireless transmission this is usually not good enough to fulfill spectrum regulations. Therefore we use a raised cosine filter with roll off factor 0.2 in our simulations to reduce the out of band spectral components. We also leave out some subcarriers at the edges of the spectrum, so that only 160 subcarriers are used. These subcarriers are within the pass-band of the raised cosine filter and are therefore not influenced by this transmit filter. The transmit filter is implemented in the digital domain. So it does not reduce the effects of nonlinear distortion caused by the analogue transmit amplifier.

Fig. 6 shows the power spectral density (PSD) of the OFDM-4FSK transmit signal for different IBOs if random binary phases are used for the subcarriers. The larger the IBO the lower the influence of nonlinear distortion and the

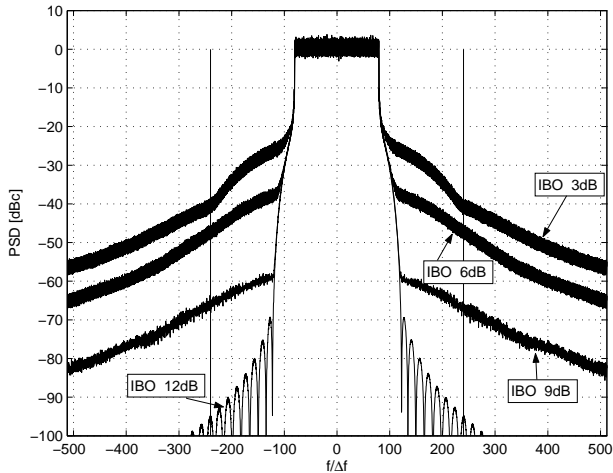


Fig. 6. PSD of the OFDM-4FSK transmit signal with nonlinear distortion and discrete random phases $\varphi_n = \{0, \pi\}$. $N = 256$, $N_{\text{used}} = 160$.

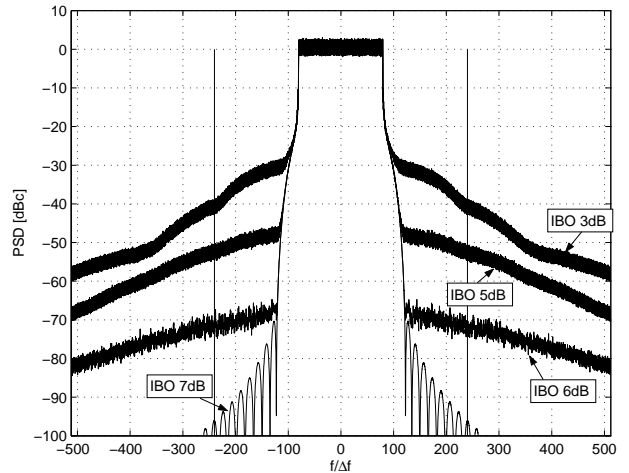


Fig. 8. PSD of the OFDM-4FSK transmit signal with nonlinear distortion using the sequential algorithm. $\varphi_n = \{0, \pi\}$, $N = 256$, $N_{\text{used}} = 160$.

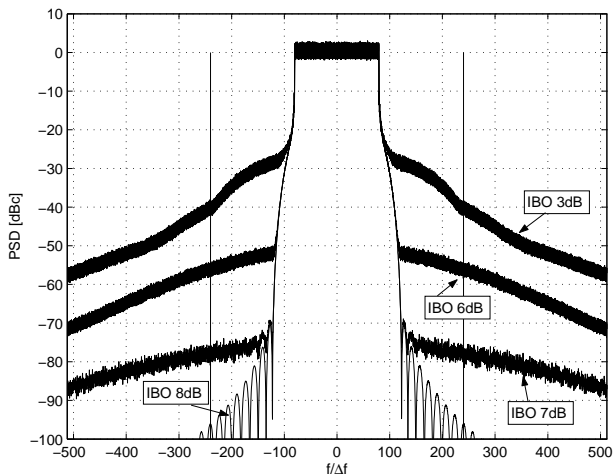


Fig. 7. PSD of the OFDM-4FSK transmit signal with nonlinear distortion using the selected mapping technique where the best of ten candidate symbols is transmitted. $\varphi_n = \{0, \pi\}$, $N = 256$, $N_{\text{used}} = 160$.

OBR. As a reference point we use the PSD at a distance of 240 subcarriers to the center frequency. For an IBO of 3dB almost all symbols are distorted and so the power at the reference point is only less than 40 dB below the power of the used carriers which is too high in many cases. If we increase the IBO to more than 9 dB only very few amplitude peaks are clipped and the power drops to -65 dBc which is tolerable in most cases. For IBO = 12 dB the transmit signal is not influenced by the nonlinearity and the shape of the spectrum is determined by the digital transmit filter. If we use the selected mapping technique to reduce the PAPR, values around -70 dBc can be obtained already with an IBO of 7 dB, as can be seen in Fig. 7. The IBO can be further reduced to 6 dB to obtain -70 dBc at the reference point if we use the sequential algorithm (Fig. 8). Simulations have shown, that using the swapping algorithm, only minor improvements regarding the IBO can be made compared to the sequential algorithm.

VI. CONCLUSION

In this paper, we compared several PAPR reduction methods which can be used for noncoherently detected OFDM-MFSK. We also presented a new algorithm which adjusts the subcarrier phases sequentially to reduce the PAPR. In general an increase in performance comes along with a higher complexity of the algorithm. The new algorithm can be seen as a trade off between performance and complexity. We also showed the influence of nonlinear distortion on the spectrum of the transmit signal.

ACKNOWLEDGMENT

This work was conducted in cooperation with TELE-FUNKEN RACOMS within the WIGWAM project, supported by the German Federal Ministry of Education and Research (BMBF).

REFERENCES

- [1] S. H. Han and J. H. Lee, "An Overview of Peak-to-Average Power Ratio Reduction Techniques for Multicarrier Transmission," *IEEE Wireless Communications*, vol. 12, pp. 56-65, April 2005.
- [2] R. van Nee and R. Prasad, *OFDM for Wireless Multimedia Communications*, Artech House Publishers, 2000.
- [3] M. Friese, "Multitone Signals with Low Crest Factor," *IEEE Transactions on Communications*, vol. 45, pp 1338-1344, October 1997.
- [4] A. E. Jones, T. A. Wilkinson, S. K. Barton, "Block Coding Scheme for Reduction on Peak to Mean Envelope Power Ratio of Multicarrier Transmission Schemes," *IEEE Electronics Letters*, vol. 30, pp 2098-2099, December 1994.
- [5] R. W. Bäuml, R. F. H. Fischer, J. B. Huber, "Reducing the Peak-to-Average Power Ratio of Multicarrier Modulation by Selected Mapping," *IEEE Electronics Letters*, vol. 32, pp 2056-2057, October 1996.
- [6] E. van der Ouderaa, J. Schoukens, J. Renneboog, "Peak Factor Minimization Using a Time-Frequency Domain Swapping Algorithm," *IEEE Transactions on Instrumentation and Measurement*, vol. 37, pp 145-147, March 1988.
- [7] S. Grob, *Modeling Nonlinearities in Communication Systems and Mitigating Their Effects*, PhD Thesis, University of Ulm, 2005. Available: http://it.e-technik.uni-ulm.de/World/Research.IT/sq_diss.chp2.pdf