# OFDM-MFSK with Differentially Encoded Phases for Robust Transmission over Fast Fading Channels

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***Abstract*— We present a new OFDM-based hybrid transmission scheme which is suitable for robust transmission in fast fading environments, where a reliable channel estimate is impossible or very difficult to obtain. Our scheme is based on the combination of noncoherently detected MFSK (M-ary frequency shift keying) and OFDM (orthogonal frequency division multiplexing). Furthermore, the subcarrier phases of this OFDM-MFSK scheme can be differentially modulated, leading to a hybrid transmission scheme where additional data can be transmitted, without affecting the robustness of the underlying noncoherently detected OFDM-MFSK. A channel estimate is not needed both for the pure OFDM-MFSK and the hybrid modulation scheme. We also consider a coded system, where the two data streams using the MFSK and the phase component are separately encoded.**

***Index Terms*— OFDM-MFSK, robust transmission, differential modulation, noncoherent detection, hybrid modulation, fast fading.**

## I. INTRODUCTION

Obtaining a reliable channel estimate is very difficult in a fast fading environment. A simpler way is to use a modulation scheme which can be detected noncoherently and therefore does not need a channel estimation at all. In this paper we analyse a system where noncoherently detected M-ary frequency shift keying (MFSK) is combined with OFDM (orthogonal frequency division multiplexing), resulting in a very simple receiver which does not need equalization and channel estimation. The major drawback of MFSK, also in combination with OFDM, is its low spectral efficiency [2]. To mitigate this, we present a new hybrid transmission method which combines OFDM-MFSK and DPSK (differential phase shift keying).

In Section II we will briefly describe our system model. Section III first describes the principles of OFDM-MFSK and then presents the new hybrid transmission scheme which is a combination of OFDM-MFSK and DPSK. Simulation results are presented in Section IV for both AWGN and a fast fading two-path channel.

## II. SYSTEM MODEL

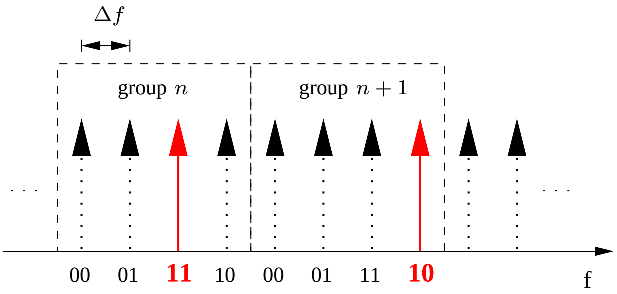

Fig. 1. Principle of OFDM-4FSK modulation: Two bits are assigned to each subcarrier using Gray coding. One out of each group of four subcarriers is taken for transmission, indicated by the solid arrow.

We use a discrete-time baseband representation of an N-tone OFDM transmission, where the time domain sample vectors $\mathbf{s}_k$ are obtained by transforming the transmit symbols $\mathbf{x}_k$ of size $N \times 1$ to the time domain, using a normalised IFFT and adding a cyclic prefix. Here $k$ stands for the symbol index. After a parallel to serial conversion we obtain the discrete-time transmit signal $s(i\Delta t)$ which is convolved with the channel impulse response $h(i\Delta t)$ and affected by AWGN:

$$g(i\Delta t) = s(i\Delta t) * h(i\Delta t) + n(i\Delta t) \tag{1}$$

In the receiver, the cyclic prefix is removed from the received signal $g(i\Delta t)$ and after serial to parallel conversion and transformation to the frequency domain via FFT, the receive symbols $\mathbf{y}_k$ are obtained.

## III. NEW HYBRID TRANSMISSION SCHEME

### *A. OFDM-MFSK*

M-ary frequency shift keying is a well known technique for robust transmission. It can be combined with OFDM by dividing the subcarriers of an OFDM symbol into groups of $M$ and applying MFSK to each of these groups. This modulation scheme allows noncoherent detection which is particularly interesting for fast fading environments because no channel estimation is needed, which might be very complex under such conditions.

Fig. 1 shows the principle of this modulation using the example of OFDM-4FSK. The subcarriers with a spacing of $\Delta f$ are grouped into blocks of four. One carrier of each group is selected for transmission whereas on the other subcarriers of the group no energy is transmitted.

Assuming only AWGN, the bit error probability for OFDM-MFSK is the same as for conventional MFSK and can be expressed as [3]

$$P_{b\text{MFSK}} = \frac{M}{2(M-1)} \sum_{n=1}^{M-1} (-1)^{n+1} \binom{M-1}{n} \cdot \frac{1}{n+1} \exp\left[-\frac{n}{n+1}\frac{log_2 M E_b}{N_0}\right]. \tag{2}$$

It is well known that for increasing $M$ this modulation scheme becomes more power efficient and approaches the Shannon Bound (also compare Fig. 4). Obviously it is possible to transmit $log_2 M$ bits per $M$ subcarriers which means that the spectral efficiency approaches zero when $M$ is increased. This low spectral efficiency is the major drawback of modulation schemes based on MFSK. A good compromise is the use of OFDM-4FSK because it has the same spectral efficiency as OFDM-2FSK of $0.5\,\text{bit}/(\text{s}\cdot\text{Hz})$ while being more robust against noise.

If we transmit over frequency selective channels caused by multipath propagation, some subcarriers can completely fade out and cause an error floor in the bit error curve. To combat this problem, channel coding in conjunction with interleaving is used. The best performance is obtained by using a soft decision detector in order to provide the decoder with a degree of reliability for each bit. A suitable metric $L_j$ for the $j$-th bit of a code symbol in a transmission with orthogonal modulation can be calculated from the components of the received vector $\mathbf{y}_{ki}$ [4]

$$L_j = \underset{i\in S_j^1}{max}|y_{ki}|^2 - \underset{i\in S_j^0}{max}|y_{ki}|^2 \tag{3}$$

$S_j^1$ denotes the subset of all component indices where the code symbols have a '1' at the $j$-th digit of the bit mapping. Accordingly there is a '0' at the $j$-th digit for $S_j^0$.

Using noncoherent detection for OFDM-MFSK, the phase of the carriers is arbitrary. This degree of freedom can be used in several ways. One possibility is to choose the subcarrier phases in such a way that the peak to average power ratio (PAPR) of each OFDM symbol is minimized [5]. An overview of PAPR reduction techniques is given in [6], which can also be applied to OFDM-MFSK. A second possibility is to use the phase to transmit additional data, thus improving the spectral efficiency of the modulation scheme. Such a scheme will be presented in the following subsection.

### *B. Hybrid OFDM-MFSK with Differentially Encoded Phases*

In this scheme the subcarrier phases of the OFDM-MFSK symbols are differentially encoded using DPSK, maintaining noncoherent detection without the need for channel estimation. A similar hybrid scheme, but in conjunction with orthogonal Walsh modulation and CDMA, was presented in [7].

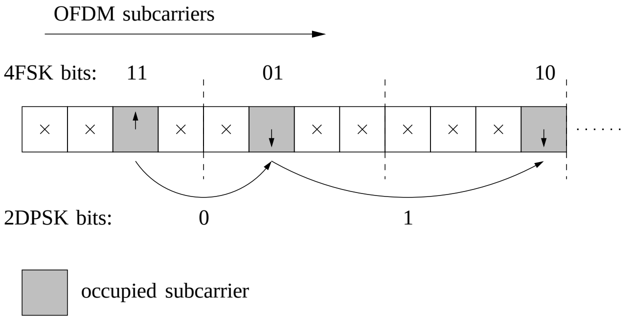

Fig. 2. OFDM-4FSK-2DPSK modulation scheme.

Basically the differential encoding of the OFDM-MFSK symbol phases can be done in two different manners. The DPSK modulation can be implemented in frequency direction from occupied subcarrier to occupied subcarrier within one OFDM symbol or in time direction from OFDM symbol to OFDM symbol within one group of subcarriers. For fast fading channels it is advantageous to perform the encoding in frequency direction, however the DPSK component is very sensitive against frequency selectivity as the occupied subcarriers can be far apart in frequency direction. The principle of this hybrid modulation scheme is shown in Fig. 2 where the example of OFDM-4FSK in conjunction with 2DPSK is taken. For this example, one out of a group of four subcarriers is occupied according to the 4FSK bits and the phase (indicated by the arrows) of the occupied subcarriers is differentially modulated between neighbouring subcarriers according to the 2DPSK bits. In the receiver the MFSK symbols are detected first, so that the occupied subcarriers are known. Then, assuming correct detection of the MFSK symbols, the differentially encoded DPSK symbols can be detected. If one of two MFSK symbols influencing a DPSK symbol was in error, the detected phase difference does not carry any information because the phase difference to an empty carrier, i.e. noise, is detected. Therefore random bits will be received. With this hybrid scheme, the spectral efficiency can be significantly increased, without influencing the noncoherent detection of the MFSK component at all. Therefore the bit error probability for the MFSK bits stays the same as for the non hybrid OFDM-MFSK (see Equ. (2)). The error probability of a DPSK bit however, depends on the error probability of the MFSK component. After some calculations it turns out, that for AWGN the probability for a correct DPSK bit using the hybrid modulation is given by

$$P'_{c\text{DPSK}} = (1 - P_{b\text{DPSK}})\,(1 - P_{s\text{MFSK}})^2 + P_{s\text{MFSK}}\left(1 - \frac{P_{s\text{MFSK}}}{2}\right) \tag{4}$$

where $P_{s\text{MFSK}}$ denotes the probability of an MFSK symbol error and $P_{b\text{DPSK}}$ denotes the probability of a DPSK bit error assuming the symbol decision for both involved MFSK symbols was correct. If we use 4FSK, the symbol error probability for the 4FSK symbols is $P_{s4\text{FSK}} = \frac{3}{2}P_{b4\text{FSK}}$. Neglecting powers of bit error probabilities, which is applicable for high SNR, this yields

$$P'_{b\text{DPSK}} = \frac{3}{2}P_{b4\text{FSK}} + P_{b\text{DPSK}} \tag{5}$$

for the probability that a DPSK bit is in error using the hybrid OFDM-4FSK-DPSK scheme.

To improve the bit error performance, channel coding including interleaving can be applied to the hybrid scheme as well. To maintain the characteristic that the robustness of the noncoherent OFDM-MFSK transmission is not influenced by the additional DPSK modulation, separate encoding of the MFSK and DPSK bit streams is reasonable. In the receiver the MFSK symbols are detected and decoded first. Then the received bits are encoded again, to obtain knowledge about the occupied subcarriers $\mathbf{c}_{occ}$. After this, the DPSK symbols can be detected and decoded as well. A block diagram of this hybrid transmission scheme including channel coding can be seen in Fig. 3.

## IV. SIMULATION RESULTS

In this section we present simulation results for both OFDM-MFSK and the hybrid modulation scheme, where OFDM-MFSK and DPSK are combined. For all simulations a two-sided noise power spectral density of $N_0/2$ is assumed, which means that white Gaussian noise with $\sigma^2 = N_0/2$ for the real and the imaginary component was added.

### A. AWGN channel

Let us start with uncoded transmission over the AWGN channel. Fig. 4 shows the BER vs. $E_b/N_0$ for noncoherently detected OFDM-MFSK and various $M$. For comparison the curve for BPSK is also added as a dashed line, but it has to be kept in mind, that BPSK has to be coherently detected and therefore needs channel knowledge. As mentioned before, for OFDM-MFSK there is a trade off between increasing power efficiency and decreasing spectral efficiency for increasing $M$. For reasons also mentioned before, $M = 4$ seems to be a good trade off and we will therefore concentrate on OFDM-4FSK in the following.

Fig. 5 shows the simulation results for the hybrid transmission scheme based on OFDM-4FSK. The BER for 4FSK-2DPSK and 4FSK-4DPSK is the total BER for both 4FSK and DPSK bits. The large gain in $E_b/N_0$ for the hybrid schemes is due to the fact, that more bits per symbol can be transmitted, therefore the energy per bit is reduced. For 4FSK-4DPSK the same spectral efficiency of $1\,\text{bit}/(\text{s}\cdot\text{Hz})$ in the uncoded case can be achieved as for BPSK. It can be seen from Fig. 5 that this hybrid modulation scheme outperforms BPSK in terms of BER for $E_b/N_0 > 4\,\text{dB}$ without the need for channel estimation.

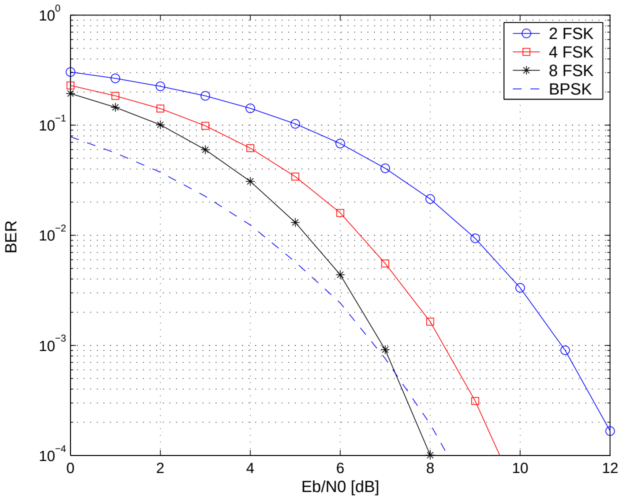

Fig. 4. BER vs. $\frac{E_b}{N_0}$ for noncoherent OFDM-FSK without channel coding for an AWGN channel; $N_g = 0$.

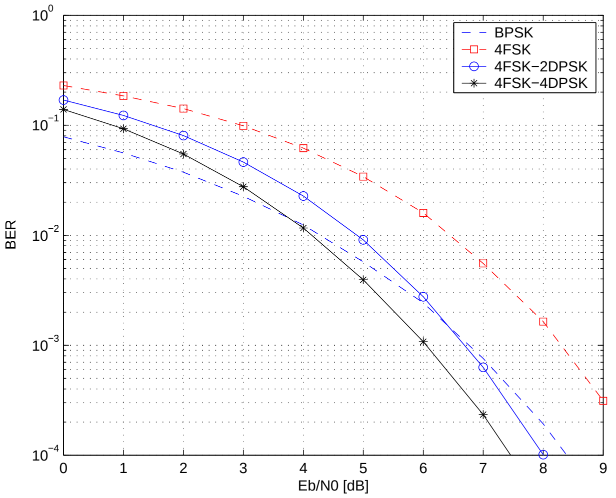

Fig. 5. BER vs. $\frac{E_b}{N_0}$ for OFDM-4FSK and hybrid OFDM-4FSK-DPSK without channel coding for an AWGN channel; $N_g = 0$.

For coded transmission, a rate 1/2 standard convolutional code with generator polynomial [133,171] and random interleaving is used for encoding both the MFSK and DPSK bits separately. In the receiver a soft input Viterbi decoder determines the received bits. For the transmission a cyclic prefix of 25% of the OFDM symbol duration is used ($N_g = 64$). Fig. 6 shows the results for coded transmission over the AWGN channel which are similar to the results for uncoded transmission. It has to be mentioned that in the case of AWGN only, the BER for the transmission with the above mentioned parameters using the hybrid schemes is dominated mostly by errors in the 4FSK transmission. The spectral efficiency could therefore be further optimized by adapting the modulation and coding methods for the DPSK transmission. However, as will become clear later, the DPSK transmission component is more sensitive against frequency selectivity or fast time variance. These effects shall be studied in the next subsection.

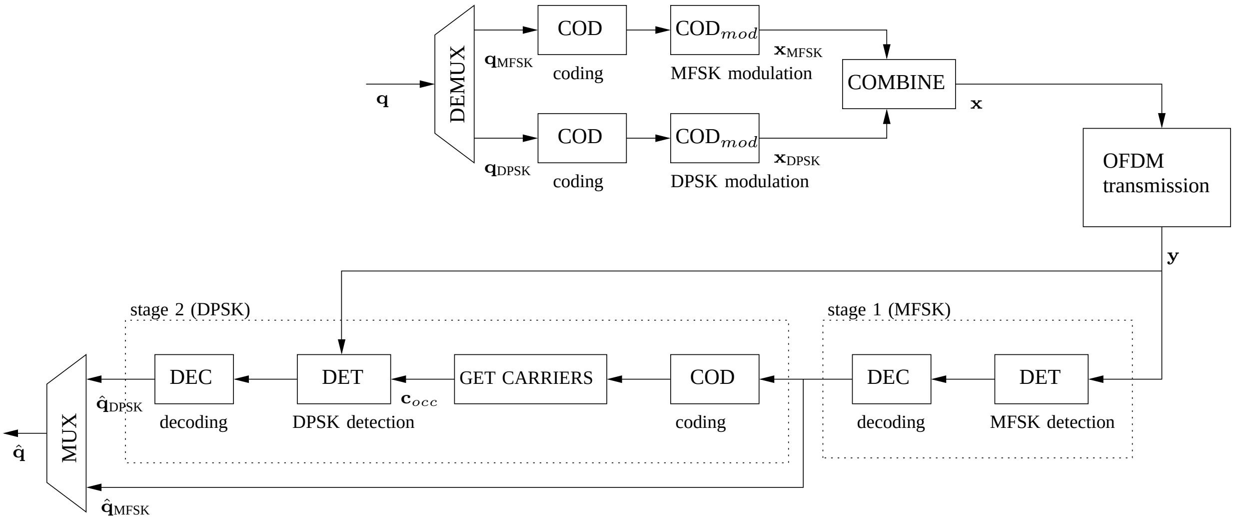

Fig. 3. Block diagram of the coded hybrid transmission scheme.

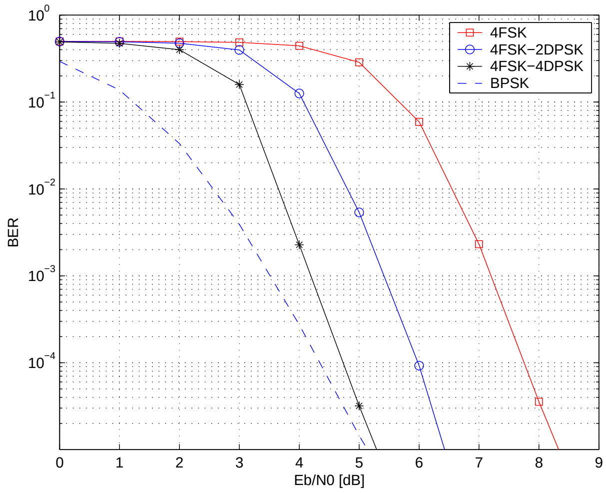

Fig. 6. BER vs. $\frac{E_b}{N_0}$ for OFDM-4FSK and hybrid OFDM-4FSK-DPSK for an AWGN channel and a rate 1/2 convolutional code ([133,171]); $N_g = 25\%$.

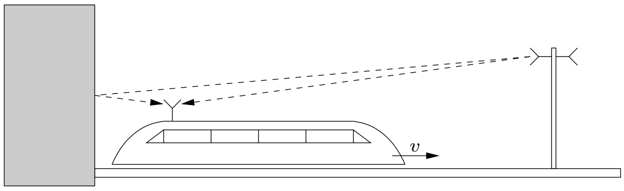

Fig. 7. Model for a two path channel in a high speed scenario, reflection at a bridge or tunnel entrance.

| | |
|---|---|
| carrier frequency | $f_c = 38\,\text{GHz}$ |
| FFT length | $N_f = 256$ |
| no. of used subcarriers | $N_{fused} = 160$ |
| subcarrier separation | $\Delta f = 312.5\,\text{kHz}$ |
| cyclic extension | $T_g = N_g \Delta t = 0.8\,\mu\text{s}$ |
| symbol duration | $T_s = (N_g + N_f)\,\Delta t = 4\,\mu\text{s}$ |

TABLE I

PARAMETERS FOR THE HIGH VELOCITY SCENARIO

### B. *Worst Case Channel Model*

Because we are interested in a robust transmission scheme we first have to define a channel model which includes the desired disturbances. As an example we take a scenario where a high speed train transmits and receives signals from a fixed base station. A worst case in this scenario would be, if in addition to a line of sight (LOS) path, a second path which is reflected behind the mobile station with low attenuation arrives at the receiver, leading to a maximum Doppler spread of $2f_d = 2f_c\frac{v}{c}$ in the received signal. Here $v$ denotes the velocity of the mobile station and $f_c$ is the carrier frequency of the OFDM system, assuming $f_c$ is much greater than the OFDM bandwidth. Such a scenario is shown in Fig. 7. The parameters used for all simulations with the time variant 2-path channel are listed in table I. Simulation results using OFDM-4FSK are plotted in Fig. 8 for different velocities of the mobile station. Furthermore it was assumed that the reflected path is not attenuated compared to the LOS path and the path delay is $t_d = 0.75\mu s$ causing very strong frequency selectivity. The simulation results show, besides the overall degradation due to the frequency selective fading, that noncoherently detected OFDM-4FSK is very robust against large Doppler spreads, occurring in fast fading environments. Even for $v = 600\,\text{km/h}$ the degradation in $\frac{E_b}{N_0}$ is less than 0.5 dB at a BER of $10^{-5}$. This robustness of OFDM-4FSK of course also holds for the 4FSK bits in the hybrid transmission scheme, however, the DPSK bits are much more sensitive against frequency selectivity, especially if the modulation is done in frequency direction. From Fig. 9 we can see that even for a path delay of $t_d = 0.075\,\mu s$ the frequency selectivity still causes a severe performance degradation in the DPSK component. For a smaller delay of $t_d = 0.03\,\mu s$ this degradation becomes less. Note that the simulations in Fig. 9 were done with a velocity of $v = 600\,\text{km/h}$ for the mobile station and DPSK modulation in frequency

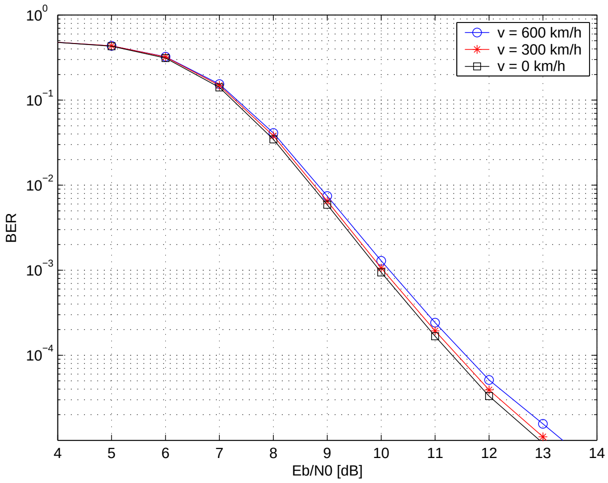

Fig. 8. BER vs. $\frac{E_b}{N_0}$ at the receiver for OFDM-4FSK with a rate 1/2 convolutional code ([133,171]) for the two path channel; $t_d = 0.75\,\mu s$.

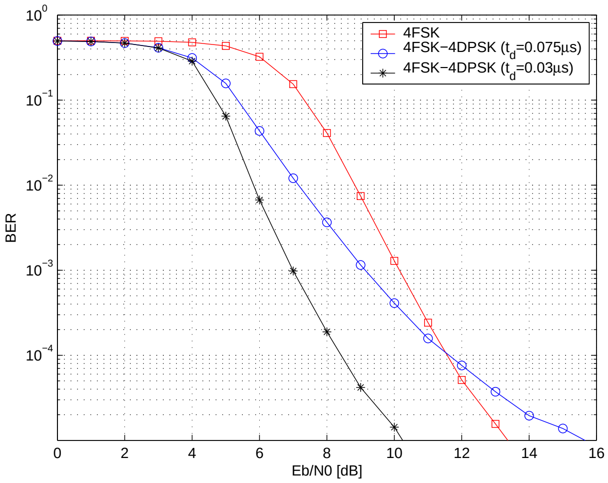

Fig. 9. Total BER vs. $\frac{E_b}{N_0}$ at the receiver for OFDM-4FSK-4DPSK with with a rate 1/2 convolutional code ([133,171]) for the two path channel; $v = 600\,\text{km/h}$.

direction, proving, that also the hybrid scheme is very robust against fast time variance. If the DPSK modulation is done in time direction, the BER is much more sensitive against fast time variance, making a transmission over the DPSK component at $v = 600\,\text{km/h}$ impossible. The sensitivity against frequency selectivity is lowered but still quite high, because different subcarriers are occupied in consecutive symbols and differentially modulated in time direction.

## V. SUMMARY AND CONCLUSION

In this contribution a new OFDM-based robust modulation scheme was presented and analysed. $M$ OFDM subcarriers are grouped together and a MFSK modulation is applied over these subcarriers. Noncoherent OFDM-MFSK detection is possible, making the scheme very robust against fast fading channels and rendering channel estimation unnecessary. To increase the spectral efficiency it is possible to differentially modulate the phases of occupied subcarriers. This additional phase modulation does not affect the noncoherent OFDM-MFSK transmission but offers additional data rate for moderate channel conditions.

## ACKNOWLEDGMENTS

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